

# Diffusion in the French Input-output Network

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Technological Evolution and Economic Growth

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# Outline

## French Input-Output Network

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

- 1 Research question
- 2 Related literature
- 3 Data and Methods
- 4 Structural Properties
- 5 Spreading Mechanisms in the input-output network
- 6 Discussion

# Motivation

## French Input-Output Network

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### Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

- Structure determines functionality and performance
- Structure of the economy can be represented by the input-output network: how sectors use output of other sectors as input to produce.
- Change in structure → Technological change due to:
  - 1 Changes in final demand of HH, Gov't, X, K,
  - 2 Changes in technology

# Research question

French  
Input-Output  
Network

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

- How to better capture linkages among sectors and how changes in structure impact the macroeconomy through these linkages?
- Are complex network measures useful tools to identify key sectors in the economy and make policy recommendations?

## Input-output Analysis:

- Interdependencies between sectors in an economy or region (Leontief, 1936,1967; Isard, 1951; Hewings et al., 1986).
- Backward and forward linkages, key sectors, and structural decomposition (Dietzenbacher and Los, 1998; Dietzenbacher, 1992; McGilvray,1977; Rasmussen,1956).

## Input-output and network analysis:

- Network properties of the economy (Slater, 1978; Blochlet et al., 2011; Xu, 2011; Wen-Qi Duan, 2012)
- Intersectoral linkages in a multisectoral macroeconomic model (Carvalho, 2010; Acemoglu et al., 2012).

## Networks and diffusion:

- spread of ideas, innovations, diseases, failures (Kinney et al., 2005; Garal et al., 2010; Lee et al., 2011).

# Data and Methods

- Input-output table for France for 2007 provided by the INSEE: 116 sectors at current prices in euros

# The Input-output Model

- The flow of products from one sector to another is captured in the interindustry transactions table  $z_{ij}$ :

		Buying sector		
		1	2	3
Selling sector	1	$z_{11}$	$z_{12}$	$z_{13}$
	2	$z_{21}$	$z_{22}$	$z_{23}$
	3	$z_{31}$	$z_{32}$	$z_{33}$

- Output of a sector,  $x_j$ , is expressed as a function of the demand for inputs,  $[a_{ij}] = [z_{ij}/x_j]$ . In matrix form:

$$x = Ax + DF \Rightarrow x = (I - A)^{-1}DF = LDF \quad (1)$$

where  $x$  is the production vector,  $A = [a_{ij}]$  is the technical coefficients matrix,  $DF$  is the final demand of the exogenous sector (HH, Gov't, exports), and  $L = [l_{ij}]$  is the Leontief inverse



# Input-output Analysis: Linkages and Key Sectors

An increase in production has two effects:

$$\Delta x = x^1 - x^0 = LDF^1 - LDF^0 = L = I + A + A^2 + \dots = (I - A)^{-1} \quad (2)$$

- **Total Backward linkages:** direct and indirect effects on demand.  
Measures the output needed to satisfy additional demand  $\Delta x \rightarrow \Delta D$

$$BL(total)_{.j} = \sum_{i=1}^n l_{ij} = (I - A)^{-1} \mathbf{1} \quad (3)$$

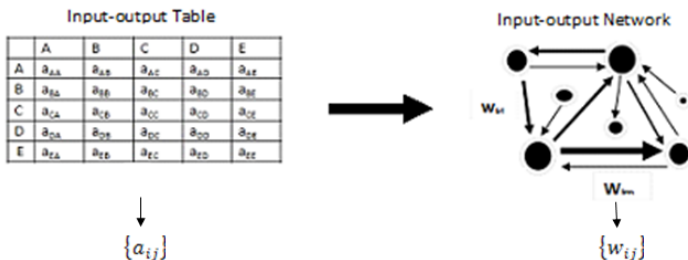
- **Total Forward linkages:** direct and indirect effects on supply.  
Measures additional output available to be used as inputs  $\Delta x \rightarrow \Delta S$

$$FL(total)_i = \sum_{j=1}^n l_{ij} = (I - A)^{-1} \mathbf{1} \quad (4)$$

- **Key Sectors:** those with both linkages above average

# The Input-Output Network

The matrix of technological coefficients gives rise to a weighted directed graph with self loops:  $A=[a_{ij}]=W=[w_{ij}]$



**Figure :** Construction of the input-output network. Nodes are sectors and weighted, directed edges represent economic relationships between sectors.

# Complex Network Analysis

## Network measures

- Measures of local centrality:
  - **In strength**: magnitude of economic relationship between a sector and its direct input suppliers

$$s^{in} = \sum_{j \neq i} a_{ji} = A^T \mathbf{1} \quad (5)$$

- **Out strength**: magnitude of economic relationship between a sector and its direct input buyers

$$s^{out} = \sum_{j \neq i} a_{ij} = A \mathbf{1} \quad (6)$$

- These measures are equivalent to the **direct linkage** measures proposed in Input-output analysis using the A matrix.

# Authority and Hub Scores

Production of sector  $j$  increases. **Two chains of effects:**

1. Demand of  $j$  for inputs increases; 2. Supply of inputs increase, but to do so sectors increase production; 3. To produce more they demand more inputs:  $D \rightarrow S \rightarrow D$ .
2. 1. Sector  $j$  increases supply of inputs; 2. Demand for inputs increase and production increases; 3. Supply of inputs increases:  $S \rightarrow D \rightarrow S$ .

$$s(j) \leftarrow \sum_{j \rightarrow i} d(i) \quad (7)$$

$$d(i) \leftarrow \sum_{i \rightarrow j} s(j) \quad (8)$$

$$\vec{s} \leftarrow A\vec{d} \Rightarrow \vec{s} \leftarrow AA^T\vec{s} \quad (9)$$

$$\vec{d} \leftarrow A^T\vec{s} \Rightarrow \vec{d} \leftarrow A^T A\vec{d} \quad (10)$$

This was a 1st round of effects. For higher order effects we iterate this process  $k \rightarrow \infty$ , this converges to the principal eigenvectors of  $AA^T$  and  $A^T A$ .

# Complex Network Analysis

French  
Input-Output  
Network

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

Measures of global centrality:

- **Authority scores:** a buyer that is pointed to by highly ranked (important) suppliers  $\Rightarrow$  higher order effects on supply

$$X = (I - \lambda A^T A)^{-1} \mathbf{1} \quad (11)$$

- **Hub scores:** a supplier that points to highly ranked (important) buyers  $\Rightarrow$  higher order effects on demand

$$Y = (I - \lambda A A^T)^{-1} \mathbf{1} \quad (12)$$

- Authority (hub) scores are the principal eigenvectors of  $A^T A$  ( $A A^T$ ).
- Mutually reinforcing relationship
- **Key sectors:** those with the highest authority score, hub score, or both.

## Structural properties of the French input-output network

# Input-Output Network, France 2007

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

<b>Characteristic of the I-O network, France 2007</b>	
Number of sectors	116
Interactions (number of edge weights > 0)	7,983
Density	0.59
Percentage of sectors with self loops	90

Figure : Topology

# Input-Output Network, France 2007

## French Input-Output Network

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### Introduction

### Data and Methods

### Results France

### Spreading mechanisms

### Summary of Results

### Discussion and Conclusions

## Input-Output Network, France 2007

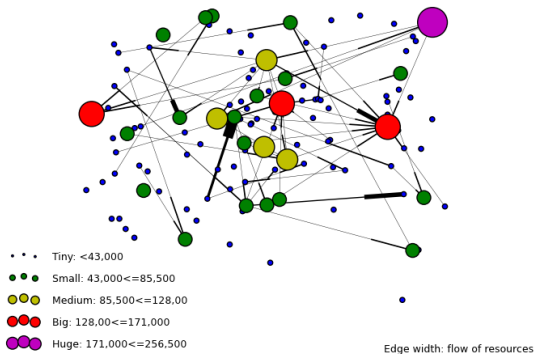


Figure : French input-output network. 25 percent largest transactions. Sectors' size and color is production level in million Euro.



# Properties of the Input-Output Network, France 2007

## French Input-Output Network

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Introduction

Data and Methods

Results  
France

Spreading mechanisms

Summary of Results

Discussion and Conclusions

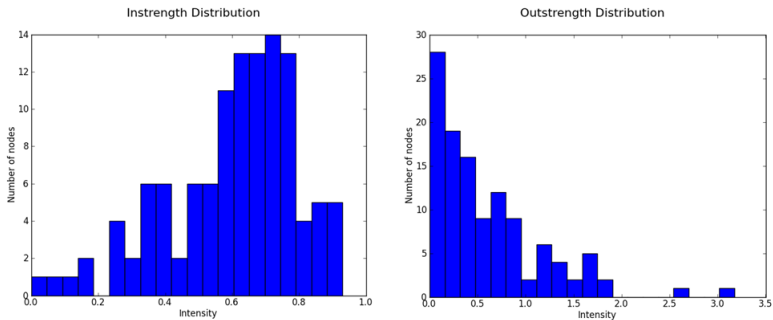


Figure : Strength Distributions

# Properties of the Input-Output Network, France 2007

French  
Input-Output  
Network

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Introduction

Data and  
Methods

Results  
**France**

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

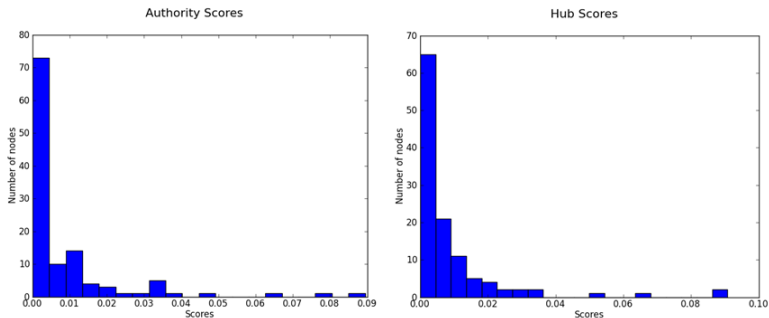


Figure : Authority and hub scores

# Key Sectors, France

French  
Input-Output  
Network

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

Key Sectors According to Standard IO Analysis					
Ranking (most to less important)	Backward linkage	Forward linkage	Linkage Key sectors	Authority Score	Hub Score
1	Aeronautics and space building	Security, cleaning and other services	Aeronautics and space building	Oil refining	<b>Metallic minerals extraction</b>
2	Automobile	Oil refining	Non-ferrous metals production	Security cleaning and other services	Oil refining
3	Non-ferrous metals production	Hydrocarbon extraction	Organic chemistry industry	Hydrocarbon extraction	Organic chemistry industry
4	Metallic minerals extraction	Financial intermediaries	Mineral chemistry industry	<b>Agriculture</b>	<b>Meat industry</b>
5	Artificial fibers	Steel industry and steel first transformation	Paper pulp and paper	Steel industry and steel first transformation	Milk industry
6	Mineral chemistry industry	Organic chemistry industry	Steel industry and steel first transformation	Organic chemistry industry	<b>Fuel production and distribution</b>
7	Reception devices fabrication	Professional services	Plastic materials transformation	Non-ferrous metals production	<b>Food industry</b>
8	Paper pulp and paper	Non-ferrous metals production	Para-chemistry	Metallic products fabrication	Steel industry and steel transformation
9	Milk industry	Business administration	Electrical material fabrication	Financial intermediaries	Reception devices fabrication
10	Steel industry and steel first transformation	Advertising and market studies	Oil refining	Advertising and market studies	<b>Motorcycle materials</b>

- **Distribution of effect of a shock:** An increase in output has effects on supply and demand which diffuse among sectors. If sector  $k$  received the shock:

$$D_k^D = 1 - \left( \sum_{j=1}^n (l_{jk} / \sum_{j=1}^n l_{jk})^2 \right) \quad (13)$$

- $0 < D < 1$
- Low  $D \rightarrow$  more concentrated effect on one or few sectors

# Diffusion of Effect and Aggregate Impact of Shock

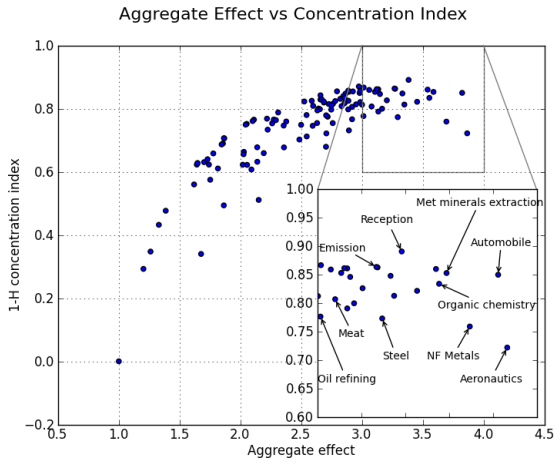


Figure : Aggregate effect on production and concentration of the effect

# Diffusion of Effect of a Targeted shock, France

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

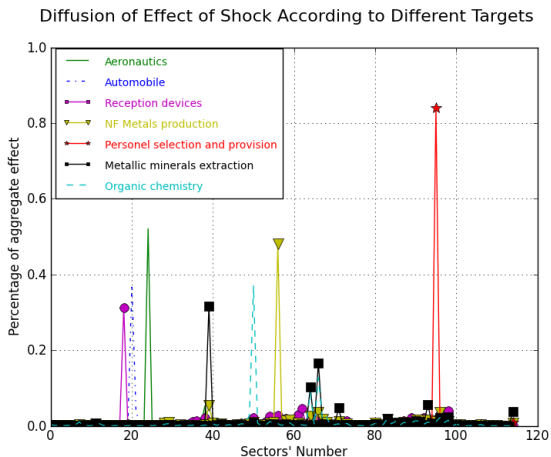


Figure : Diffusion of aggregate effect among sectors. Peaks show a high concentration of the effect.

# Spreading Mechanisms

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Introduction

Data and  
Methods

Results  
France

**Spreading  
mechanisms**

Summary of  
Results

Discussion  
and  
Conclusions

Avalanches: number of sectors affected by a propagating shock

- Shock on final demand
- Shock on technology

# Spreading Mechanisms in IO model

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

**IO Model:** how is a shock on final demand spread throughout the economy? Sector  $j$  is part of an avalanche triggered by sector  $i$  if:

$$effect_j = (x_j^i - x_j) / \sum_{j=1}^n x_j^i = l_{ji} / \sum_{j=1}^n l_{ji} > threshold \quad (14)$$

where  $x_j^i$  is production of  $j$  after a shock on final demand of  $i$ ,  $x_j$  is original production of  $j$ , and  $l_{ji}$  is the element of the  $j$ th row and  $i$ th column of the  $L$  matrix (IO Model).

- normalized shock = 1 ( a unit increase in final demand) and  $A$  matrix remains fixed



# Avalanche Size with IO Model, France

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

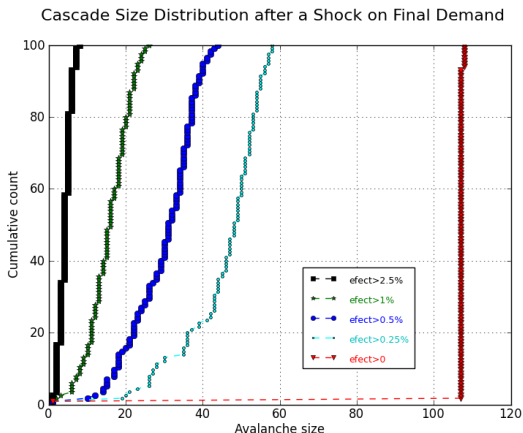


Figure : Avalanche size distribution according to different thresholds. Largest avalanches triggered by home appliances, weapons and ammunitions, and automobile equipment

# A shock on technology

**Model 1.** How is a shock on technology spread from one sector to another? (Kinney et al., 2005; Lee et al., 2011)

- Node attribute: capacity  $x_i$  = each sector's production
- Edge attribute: weight  $w_{ij} = z_{ij}$  economic transaction
- FD fixed and shock =  $f * w_{ij}$  and is the same for all sectors
- A shock hits sector  $i$ . As a result, the supply and demand of inputs of sector  $i$  decrease by a fraction  $f$ .

$$w_{ij}^* = (1 - f)w_{ij}, 0 < f < 1 \quad (15)$$

- The shock spreads to  $j$  if:

$$\sum_{k \in N(j)} (w_{jk} + w_{kj}) > (t/f) * x_j \quad (16)$$

where  $t$ : capacity threshold,  $0 < t < 1$

# Avalanche Size with Model 1, France

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Input-Output  
Network

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

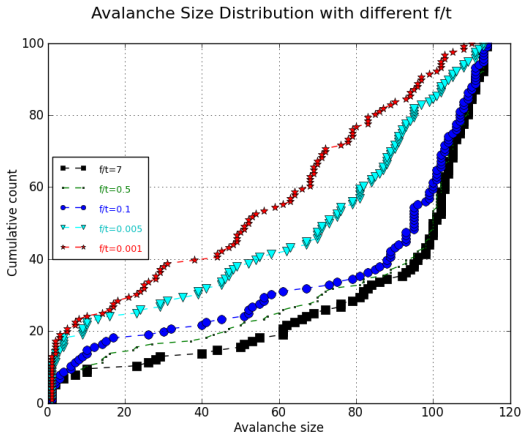


Figure : Avalanche size distribution according to different  $f/t$ .  
Largest avalanches triggered by financial, oil refining and construction

# A shock on technology

French  
Input-Output  
Network

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Alatrste

Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

**Model 2.** A sector gets hit by a shock, then, its input supply and demand decrease by a fraction  $f$ . Following, production level gets updated according to the input-output model ( $w_{ij}^* = (1 - f)w_{ij} \rightarrow a_{ij}^* = z_{ij}^*/x_j$ ).

$$x^* = (1 - A^*)^{-1}DF \quad (17)$$

Finally, the shock spreads to other sectors according to the same rule: if sector  $i$  received the original shock, for  $j$  in the neighbourhood of  $i$ , if:

$$\sum_{k \in N(j)} (w_{jk} + w_{kj}) > (t/f) * x_j \quad (18)$$

the shock spreads to  $j$ .

# Avalanche Size with Model 2, France

French  
Input-Output  
Network

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

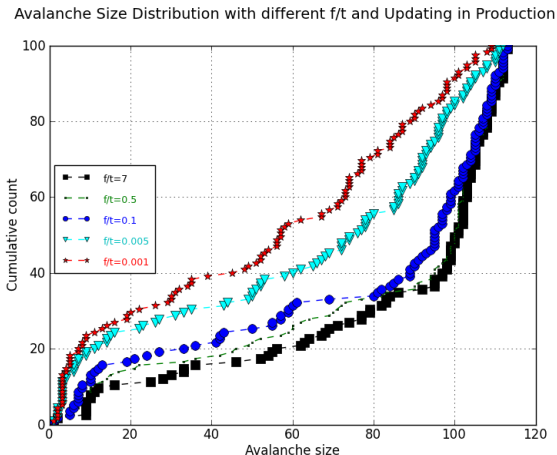


Figure : Avalanche size distribution according to different  $f/t$  and updating in production. Largest avalanches triggered by financial, oil refining, and electricity

# Summary of Results

French  
Input-Output  
Network

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

## Structural properties

- Asymmetrical structure where sectors have different roles and are organized in a hierarchy which allows the identification of key sectors.
- Sectors with the widest diffusion of effect and a high aggregate effect on production have the highest authority and/or hub scores but not necessarily the highest (IO) linkages.
- Authority scores: higher order effects on supply of inputs.
- Hub scores: higher order effects on intermediate demand of inputs.

# Results on Spreading Mechanisms

French  
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Network

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Introduction

Data and  
Methods

Results  
France

Spreading  
mechanisms

Summary of  
Results

Discussion  
and  
Conclusions

- **IO model:** every sector triggers large avalanches of same size (107 sectors)
- **Model 1:** heterogeneous but predominantly large avalanches
- **Model 2:**  $P(A > 85\%)$  decreased, but  $P(A < 50\%)$  increased → more small-med avalanches;  $P(50\% < A < 85\%)$  mixed results
- Capacity threshold and weights distribution are stronger determinants of spreading mechanisms.
- Sectors that triggered the largest avalanches using model 1 are the same as using model 2 but different from IO model: financial, oil refining, and construction.

# Discussion

- Authority and hub scores provide additional information taking into account higher order effects on supply and demand → ranking of sectors.
- Identification of key sectors in the economy leads to policy recommendations:
  - Good hubs that have high aggregate effect and good diffusion of a shock on FD
  - Triggers of large avalanches or good spreaders of a shock on technology
- Advantages of applying network analysis: spreading of a shock on technology different from spreading of a shock on final demand → beyond Input-Output model
- Analysing the structure of the economy and how it reacts to shocks as a first step to model its evolution.



Thank you for your attention!