Pakes (1986), "Patents as Options"

1 The Model

- Output of innovative activity is random. Strong incentives exist to patent early, before the value of an invention is known. Patents are renewed in order to maintain the option of future profits.

$$V(a) = \max \left\{ 0, \underset{\substack{\text{current} \\ \text{return}}}{r_a} - \underset{\substack{\text{cost of} \\ \text{renewal}}}{c_a} - \underset{\substack{\text{discounted expected} \\ \text{value of } V(a+1)}}{\beta E[V(a+1)|\Omega_a]} \right\}$$

- Assumptions

  1. Markov assumption: Distribution of r tomorrow depends only on r today, as well as the age of the patent

     Note that the distribution is non-staionary

     Also, distribution is non-decreasing in r (in expectation, higher returns today imply higher returns tomorrow) and non-increasing in a ("fishing-out" assumption)
  2. Renewal feee schedule does not change. Also, renewal fees are nondecreasing in age.
  3. A "regularity condition that insures finiteness and continuity of the value function"

- Under these assumptions, eq. 2.1 can be written as

$$V(a,r) = \max\{0, \beta E[V(a+1)|r,a] - c_a\}$$

where

$$E[V(a+1)|r,a] \equiv \int_{R^+} V(a+1,z)\, G(dz|r,a)$$

$a = 1, \ldots, L$, and L is the maximum length of the patents, which varies by country. Substituting yields

$$V(a,r) = \max\left\{0, \beta \int_{R^+} V(a+1,z)\, G(dz|r,a) - c_a\right\}$$

- Lemma 1 states that the vlalue of the option, $E\left[V\left(a+1\right)|r,a\right]$ is continuous and nondecreasing in r, and nonincreasing in a.

  How is this proved? By using the corollary to the contraction mapping theory (can discuss with you, if you like).

- The fact that renewal fees are increasing in age, while the option value is decreasing, implies that cutoffs are increasing in age.

2 Matching the model to data

See pp. 762-763 and 765-767.

3 Modeling the distribution of returns

- The conditional distribution of r_{a+1} is defined by

  $$r_{a+1}=\begin{cases} 0 & \text{with probability } \exp\left(\theta r_a\right) \text{ [patent is useless]} \\ \max\left\{\delta r_a, z\right\} & \text{with probability } \exp\left(1-\theta r_a\right) \text{ [either stay the same or improve]} \end{cases}$$

  where the density of z, $q_a\left(z\right)$, is a two-parameter exponential, that is

  $$q_a\left(z\right)=\sigma_a^{-1}\exp\left[-\left(\gamma+z\right)/\sigma_a\right]$$

  and

  $$\sigma_a=\phi^{a-1}\sigma \text{ for } a=1,\ldots,L-1$$

- Initial returns distribute lognormally

  $$\log r_1\sim\eta\left(\mu,\sigma_R\right)$$

4 Simulation

- Stages

  1. Fix the values of the parameter vector

2. That implies a distribution of r
3. If $r_1 > \bar{r}$, then continue
4. Crank the distribution forward on year.

- Solving for the cutoffs

1. In the final year

$$V(r,T) = \begin{cases} r_T - c_T & r_T > c_T \\ 0 & \text{otherwise} \end{cases}$$

2. The previous year

$$V(r,T-1) = \max\left\{0, r_{T-1} - c_{T-1} + \beta \int V(r,T)\, p(r_T | r_{T-1})\right\}$$